University of Arizona
Student Satellite Project

Technical Note

Doc. no.: GNC-016
Revision no.: 1.0
Status: Draft

Subject: Detumbling of the UASat **Date:** April 10, 2000

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Revision history:

Revision 2.0: Minor Changes	Mar. 08, 2000
Revision 1.0: Started Initial Draft	Sept. 15, 1999

1. Document Overview

This document describes the control laws for detumbling of the UASat.

2. Requirements

The purpose for the detumbling of the satellite is to prevent the satellite from becoming inoperable due to a loss of control. The satellite must be able to correct itself and regain control when it is ejected from the space shuttle or if an object hits it. The detumbling system activates when a critical point of 0.9 kg-m^2/s of momentum in any direction is reached and begins to regain control of the satellite via the b dot control law. This law operates by reducing the satellite's kinetic energy to zero, which in turn slows down the satellite until control is regained.

3. Descriptions/Designs/Discussion

3.1 Nomenclature

$\boldsymbol{\tau}^{\mathrm{scf}}$:Torque acting on the satellite core
$\boldsymbol{\omega}^{\mathrm{scf}}$:Angular velocity of the satellite core
$\boldsymbol{\mu}^{\mathrm{scf}}$:Net magnetic dipole moment generated by the torque rods
c :Proportional gain of the B-dot control law
t_{s} :Sampling time, time between two samples
$\mathbf{B}^{\mathrm{scf}}$:Earth magnetic field
$\widehat{\mathbf{B}}^{\mathrm{scf}}$:Estimate of the Earth magnetic field

$\dot{\mathbf{B}}^{\text{scf}}$:Time derivative of the Earth magnetic field
$\hat{\dot{\mathbf{B}}}^{\text{scf}}$:Estimate of the time derivative of the Earth magnetic field
ω_{c} :User selectable cut-off frequency of the state variable filter

3.2 Notation and Coordinate Frames

This section introduces the notation and symbols used in this technote. Furthermore all reference frames used in this paper will be defined.

3.2.1 Notation

Vectors will be denoted by lowercase boldface letters. Matrices will be denoted by uppercase boldface letters. Scalars are denoted by italic lowercase letters. For example $\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$ means that the matrix $\mathbf{A}$ multiplied by vector $\mathbf{x}$ equals a scalar λ times the same vector $\mathbf{x}$.

In general, let $\tilde{\mathbf{a}}$ denote the *cross-product matrix* of vector $\mathbf{a}$, the 3×3 skew-symmetric matrix such that $\tilde{\mathbf{a}}\mathbf{b} = \mathbf{a} \times \mathbf{b}$ for any vector $\mathbf{b}$. Algebraically, $\tilde{\mathbf{a}}$ is given by

$$\tilde{\mathbf{a}} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{1}$$

The coordinate frame of reference is denoted by a superscript. For example, the vector $\mathbf{x}^{\text{ECI}}$ is given in coordinates of the ECI-frame.

3.2.2 Definition of Reference Frames

The following reference frames are used in this technote.

3.2.2.1 Spacecraft Frame (SCF)

The origin of the spacecraft fixed SCF frame is at the center of mass of the satellite. The z-axis points along the bore-sight of the telescope axis. The x-axis is perpendicular to the z-axis and points to the center of the first side panel. The y-axis is chosen such that a right-hand orthonormal reference frame is formed. The reference frame is shown in Fig. 1.

3.3 Derivation of the B-dot control law

The goal of detumbling is to reduce the kinetic energy of the satellite core to zero. The kinetic energy decreases if the dot product of the angular velocity and torque is negative

$$(\boldsymbol{\omega}^{\text{scf}})^t \boldsymbol{\tau}^{\text{scf}} < 0 \tag{2}$$

The torque produced by the torque rods on the satellite core is

$$\boldsymbol{\tau}^{\text{scf}} = \tilde{\boldsymbol{\mu}}^{\text{scf}} \mathbf{B}^{\text{scf}} \tag{3}$$

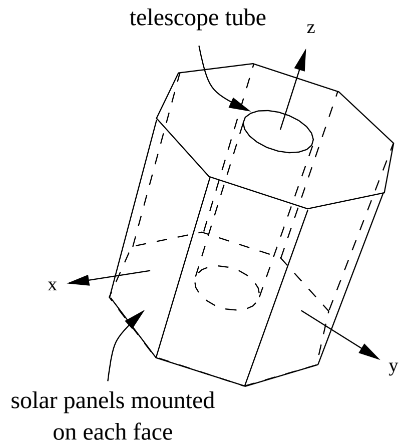

Figure 1: Geometry of the UASat and the spacecraft fixed frame

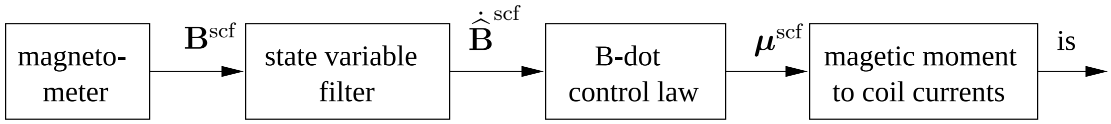

Figure 2: Block diagram of B-dot control law

Substitution of this expression for the torque in (2) yields

$$(\boldsymbol{\omega}^{\text{scf}})^t(\tilde{\boldsymbol{\mu}}^{\text{scf}}\mathbf{B}^{\text{scf}}) < 0 \quad \Leftrightarrow \quad -(\boldsymbol{\omega}^{\text{scf}})^t(\tilde{\mathbf{B}}^{\text{scf}}\boldsymbol{\mu}^{\text{scf}}) < 0 \tag{4}$$

were we used that in general $\tilde{\mathbf{a}}\mathbf{b} = -\tilde{\mathbf{b}}\mathbf{a}$. Also, in general $\mathbf{a}^t(\tilde{\mathbf{b}}\mathbf{c}) = \mathbf{c}^t(\tilde{\mathbf{a}}\mathbf{b})$. Hence

$$(\boldsymbol{\mu}^{\text{scf}})^t(\tilde{\boldsymbol{\omega}}^{\text{scf}}\mathbf{B}^{\text{scf}}) > 0 \tag{5}$$

A suitable choice for the net magnetic dipole moment that satisfies inequality (5) if $\mathbf{B}^{\text{scf}}$ and $\boldsymbol{\omega}^{\text{scf}}$ are not alligned is

$$\boldsymbol{\mu}^{\text{scf}} = c\tilde{\boldsymbol{\omega}}^{\text{scf}}\mathbf{B}^{\text{scf}} \tag{6}$$

where $c > 0$ is a user selectable gain. A larger c will result in a faster decrease of the kinematic energy. Assuming that the change in the magnetic field $\mathbf{B}^{\text{scf}}$ is mainly due to rotation of the satellite ($\dot{\mathbf{B}}^{\text{scf}} \approx \tilde{\boldsymbol{\omega}}^{\text{scf}}\mathbf{B}^{\text{scf}}$) we obtain the simple B-dot control law

$$\boldsymbol{\mu}^{\text{scf}} = c\dot{\mathbf{B}}^{\text{scf}} \tag{7}$$

In practice an estimate $\hat{\dot{\mathbf{B}}}^{\text{scf}}$ of the derivative $\dot{\mathbf{B}}^{\text{scf}}$ will be used in the control law

$$\boldsymbol{\mu}^{\text{scf}} = c\hat{\dot{\mathbf{B}}}^{\text{scf}} \tag{8}$$

3.4 Implementation

3.4.1 State variable filter

A state variable filter will be used to estimate the derivative of the Earth magnetic field. A block diagram of the filter is shown in Figure 3. The transfer function of the filter in the

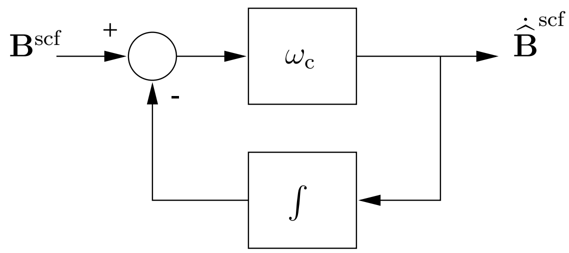

Figure 3: Block diagram state variable filter

Laplace domain easily derived. It follows from the block diagram that

$$\dot{\widehat{\mathbf{B}}}^{\text{scf}}(s) = \omega_\text{c}(\mathbf{B}^{\text{scf}}(s) - \frac{1}{s}\dot{\widehat{\mathbf{B}}}^{\text{scf}}(s)) \tag{9}$$

where s is the Laplace variable. Hence the transfer function is

$$\frac{\dot{\widehat{\mathbf{B}}}^{\text{scf}}}{\mathbf{B}^{\text{scf}}}(s) = \frac{s\omega_\text{c}}{s + \omega_\text{c}} \tag{10}$$

where ω_c is the user selectable cut-off frequency. For low frequencies ($s \ll \omega_\text{c}$) the transfer function approximates s which corresponds to a pure differentiation. For large frequencies ($s \gg \omega_\text{c}$) the transfer function approximates ω_c which is just a gain.

The cut-off frequency ω_c should be chosen approximately 5 to 10 times as high as the fastest change of $\mathbf{B}^{\text{scf}}$. This is determined by the maximum spin rate of the satellite we expect to see. Assuming that the change in the magnetic field $\mathbf{B}^{\text{scf}}$ is mainly due to rotation of the satellite ($\dot{\mathbf{B}}^{\text{scf}} \approx \tilde{\boldsymbol{\omega}}^{\text{scf}}\mathbf{B}^{\text{scf}}$) the maximum change is determined by the maximum spin rate of the satellite and the maximum magnitude of the magnetic field.

In practice the filter will be implemented using a computer and the integration will be performed numerically. The forward Euler integration rule is

$$\widehat{\mathbf{B}}^{\text{scf}}(k+1) = \widehat{\mathbf{B}}^{\text{scf}}(k) + \dot{\widehat{\mathbf{B}}}^{\text{scf}}(k)t_\text{s} \tag{11}$$

where t_s is the sampling time. Using this numerical integration scheme the equations to propagate the state of the filter from time kt_s to time $(k+1)t_\text{s}$ are

$$\mathbf{e}(k) = \mathbf{B}^{\text{scf}}(k) - \widehat{\mathbf{B}}^{\text{scf}}(k) \tag{12}$$

$$\dot{\widehat{\mathbf{B}}}^{\text{scf}}(k) = \omega_\text{c}\mathbf{e}(k) \tag{13}$$

$$\widehat{\mathbf{B}}^{\text{scf}}(k+1) = \widehat{\mathbf{B}}^{\text{scf}}(k) + \dot{\widehat{\mathbf{B}}}^{\text{scf}}(k)t_\text{s} \tag{14}$$

4. Current Status

There is currently a working simulation of the satellite being hit requiring it to go into detumbling mode and, subsequently, momentum dumping[1] once the angular momentum has decreased sufficiently.

5. References

[1] Gregg Radtke. *Momentum Dumping.* technote GNC-015, Student Satellite Project, University of Arizona.